# On a Characteristic of Data Transmission In Bus Network

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*Abstract***—Bus network is a primary topology in local area network and the network had a problem of data collision on the bus which influenced the efficiency of data transmission. Recently, using switching-hub that establishes a link from a sending station to a receiving station, the number of data collisions extremely decreased. However, in the case that the data buffer of a receiving station is full or the receiving station is sending or receiving data, a sending station has to wait for sending data. This waiting time (stand-by time) influences an efficiency of data transmission. In this paper, a characteristic of data transmission with the stand-by time in bus network is clarified. Taking note of one station with a sending data buffer, the link probability that the link from the station to a receiving station will be established is defined as constant. The station connected to the bus is modeled as a queue with stand-by time, and then state equations of the probability of the number of waiting data at the ready time of sending are expressed. From the equations, probabilities of the number of waiting data, mean of the number of waiting data in the sending buffer and mean waiting time which is the time from the data arrival time at the sending buffer to the beginning time of sending are derived. The calculated values are illustrated with the link probabilities in several cases, and then the influences of the link probability to data transmission are shown.** 

*Index Terms***—Bus network, link probability, the number of waiting data, mean waiting time.**

### I. INTRODUCTION

In bus network which is a primary topology for LAN (Local Area Network), stations are connected to a bus and send or receive data each other through the bus. In the protocol CSMA/CD (Carrier Sense Multiple Access / Collision Detection) used in the bus network, a sending station has to wait for sending data in the case that a data collision occurs on the bus. However, recently, stations are connected to a switching hub which establishes a link from a sending station to a receiving station, so that the number of data collisions extremely decreased.

In this bus network with a switching hub, a sending station also has to wait for sending in the case that a receiving buffer at a receiving station is full or the receiving station is sending or receiving data.

In the studies for bus network, the proposal of MAC (Media Access Control) protocol [1], the simulation of MAC protocol for the performance evaluation [2] and the proposal of the dynamic bandwidth allocation [3] for Ethernet PON

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(Passive Optical Network) were presented. And, the time synchronization on Gb/s Ethernet [4] which is necessary for the wide distributed system, the host configuration mechanism to network in Ethernet and wireless LAN [5] are studied. For application of Ethernet, an application method in industry [6], application to production factory [7], the performance of industrial Ethernet [8] and the performance of TDMA(Time Division Multiple Access) based network [9] are reported.

However, the characteristics of data transmission in the bus network(probabilities of the number of waiting data, mean of the number of waiting data and mean waiting time) in this paper are not cleared, and simulations for a bus network with CSMA/CD [10], [11] are only studied.

In this paper, the probability that a link from a sending station to a receiving station can be established (called as link probability) is defined as constant, and a characteristic of transmission for sending data in bus network is clarified. Taking note of one station among stations connected to a bus, probabilities of the number of waiting data, mean of the number of waiting data in the sending buffer and mean waiting time from the arrival time at the sending buffer to the beginning time of sending [12], [13] are derived. Furthermore, some calculated values are illustrated, and then characteristics are considered.

#### II. MODELING OF BUS NETWORK

The bus network shown in Fig. 1 is analyzed.



In the network, each station has an infinite buffer called as sending buffer at which messages arrive in Poisson process. The arrival rate of messages is denoted by  $\lambda$ . The station sends the first datum in the sending buffer to a receiving station in the case that the link to the receiving station is established. If not established, the station has a little waiting time (called as stand-by time). In the case that there is no data in the sending buffer, the station immediately has stand-by time. Defining the link probability as *q*, the flowchart of sending data is shown in Fig. 2.

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Fig. 2. Flowchart of sending data.

The sending station is modeled as a queue with stand-by time [14]. This is shown in Fig. 3.

Fig. 3. Queue with stand-by time.

I define that one transmission requires arbitrary distributed transmission time. The distribution function (DF) of the transmission time for the station is denoted by  $H(t)$ . The Laplace-Stieltjes Transformation (LST) of the DF, the mean of the DF and the second moment of the DF are denoted by  $H^*(s)$ , *h* and  $h^{(2)}$ , respectively. The stand-by time is arbitrary distributed, and the DF of the stand-by time, the LST of the DF, the mean of the DF and the second moment of the DF are denoted by  $H_c(t)$ ,  $H_c^*(s)$ ,  $h_c$  and  $h_c^{(2)}$ , respectively. denote the sending buffer with a sending buffer with its contribution of the sending buffer with a sending buffer with  $\frac{1}{\sqrt{2}}P_{\text{eff}}(x) = \frac{1}{2}P_{\text{eff}}(x)$ . (1)  $\frac{1}{\sqrt{2}}P_{\text{eff}}(x) = \frac{1}{2}P_{\text{eff}}(x) = \frac{1}{2}P_{\text{eff}}(x$ 

## III. PROBABILITY OF THE NUMBER OF WAITING DATA, MEAN OF THE NUMBER OF WAITING DATA, MEAN WAITING TIME

#### *A. Balance Equation*

Let the probability that the number of waiting data in the station is n at the ready time of sending be  $P_n$ . In steady state, the following balance equations are obtained.

(i) *n'*≧1

$$
P_{n'} = q \int_{0}^{\infty} \sum_{n=1}^{n'+1} \alpha(n'-n+1,t) P_n dH(t)
$$
  
+ 
$$
(1-q) \int_{0}^{\infty} \sum_{n=1}^{n'} \alpha(n'-n,t) P_n dH_c(t)
$$
 (1a)  
+ 
$$
\int_{0}^{\infty} \alpha(n',t) P_0 dH_c(t)
$$

(ii) *n'*=0

$$
P_0 = q \int_0^\infty \alpha(0, t) P_1 dH(t)
$$
  
+ 
$$
\int_0^\infty \alpha(0, t) P_0 dH_c(t)
$$
 (1b)

In the above equations,  $\alpha(n', t)$  is the probability that *n* 

$$
\alpha(n,t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!} \tag{2}
$$

The equation which is took  $\overline{n'=1}$ ∞  $\sum_{n'=1}^{\infty}$  to equation (1a) after

multiplying  $x^{n'}$  in both sides is added to equation (1b), and

then the following equation is obtained.  
\n
$$
\sum_{n'=0}^{\infty} P_{n'} x^{n'} = q \int_{0}^{\infty} \sum_{n'=0}^{\infty} \sum_{n=1}^{n'+1} \alpha(n'-n+1,t) P_{n} x^{n'} dH(t)
$$
\n
$$
+(1-q) \int_{0}^{\infty} \sum_{n'=1}^{\infty} \sum_{n=1}^{n'} \alpha(n'-n,t) P_{n} x^{n'} dH_{c}(t)
$$
\n
$$
+ \int_{0}^{\infty} \sum_{n'=0}^{\infty} \alpha(n',t) P_{0} dH_{c}(t)
$$
\n(3)

And, let the generating function (GF) be defined as follows.

$$
G(x) = \sum_{n=1}^{\infty} P_n x^n
$$
 (4)

Transforming equation (3) into GF equation,  $G(x)$  is obtained.

$$
G(x) = \frac{\left\{H_c^*(z) - 1\right\} P_0}{1 - q \frac{1}{x} H^*(z) - (1 - q) H_c^*(z)}\tag{5}
$$

In equation (5),  $H^*(z)$  and  $H_c^*(z)$  are the LST of  $H(t)$  and  $H_c(t)$ , respectively, and  $z = \lambda(1-x)$ .

#### *B. Probabilities of the Number of Waiting Data*

When  $x = 1$  in equation (5), the following equation is obtained.

$$
\{q - q\lambda h - (1 - q)\lambda h_c\} G(1) - \lambda h_c P_0 = 0 \qquad (6a)
$$

In equation (6a), *h* and  $h_c$  are mean of  $H(t)$  and  $H_c(t)$ , respectively.

Because the sum of all probabilities is 1, equation (6b) is obtained.

$$
G(1) + P_0 = 1 \tag{6b}
$$

From equation (6a) and (6b), the following probabilities are derived.

$$
G(1) = \sum_{n=1}^{\infty} P_n = \frac{\lambda h_c}{q(1 - \lambda h + \lambda h_c)}
$$
 (7a)

$$
P_0 = \frac{q(1 - \lambda h + \lambda h_c) - \lambda h_c}{q(1 - \lambda h + \lambda h_c)}
$$
 (7b)

*G*(1) is the probability that the number of waiting data is 1 or more at the ready time of sending, and  $P_0$  is the probability that there is no waiting data.

Fig. 4 shows the relation of arrival ratio  $\lambda$  and the

probability  $G(1)$  in the case that  $h=5$ ,  $h_c = 1$ .



As  $\lambda$  increases, the probability  $G(1)$  increases. This means that the number of waiting data increases so that one or more data arrive at the sending buffer while sending a datum in the case that  $\lambda$  is high. In the region of low  $\lambda$ , the change of  $\lambda$  influences  $G(1)$  significantly. And, the probability *G*(1) increases as the link probability *q* decreases. This means that the station cannot send data because the link cannot be established, so that the number of waiting data increases.

### *C. Mean of the Number of Waiting Data*

From equation (4), mean of the number of waiting data in the sending buffer is  $G'(1)$ . Again,  $G(x)$  in equation (5) is expressed by equation (8), (9a) and (9b).

$$
G(x) = \frac{A(x)}{B(x)}\tag{8}
$$

$$
A(x) = \{H_c^*(z) - 1\} P_0
$$
 (9a)

$$
B(x) = 1 - q\frac{1}{x}H^*(z) - (1 - q)H_c^*(z)
$$
 (9b)

When  $x = 1$  after differentiating equation (8), the following equation is obtained.

$$
G'(1) = \frac{A''(1)B'(1) - A'(1)B''(1)}{2{B'(1)}^2}
$$
 (10)

where

$$
A'(1) = \lambda h_c P_0 \tag{11a}
$$

$$
B'(1) = q - q\lambda h - (1 - q)\lambda h_c \tag{11b}
$$

and

$$
A''(1) = \lambda^2 h_c^{(2)} P_0 \tag{12a}
$$

$$
A^{n}(1) = \lambda^{2} h_{c}^{2} P_{0}
$$
 (12a)  

$$
B''(1) = -2q + 2q \lambda h - q \lambda^{2} h^{(2)} - (1 - q) \lambda^{2} h_{c}^{(2)}
$$
 (12b)

 $A'$ 

 $h^{(2)}$  and  $h_c^{(2)}$  are the second moments of the DF of H(*t*) and  $H_c(t)$ , respectively.

Fig. 5 shows the relation of arrival ratio  $\lambda$  and the mean of

the number of waiting data  $G'(1)$ .



 $(h = 5, h<sub>c</sub> = 1, uniform distribution).$ 

In the above figure,  $h=5$ ,  $h_c=1$ , and  $H(t)$  and  $H_c(t)$  obey uniform distributions. In the region of low  $\lambda$ , the change of  $\lambda$  influences  $G'(1)$  significantly without the link probability *q*, and in the case of  $q = 0.1$ ,  $G'(1)$  is also influenced by  $\lambda$  in the region of high  $\lambda$ . There are large differences in  $G'(1)$  between  $q = 0.1$  and others, and small differences in  $G'(1)$  in the case of  $q = 0.3$  and more.

Fig. 6 shows a relation of arrival ratio  $\lambda$  and the mean of the number of waiting data  $G'(1)$  in the case that  $H(t)$  and  $H_c(t)$  obey exponential distributions.



Fig. 6 is similar to Fig. 5, so that  $G'(1)$  is not influenced by the distributions of  $H(t)$  and  $H_c(t)$  very much.

#### *D. Mean Waiting Time*

The probability that the number of waiting data at the end of sending data is  $n'$  is denoted by  $R_n$ , and then the following equation is obtained.

$$
R_{n'} = \frac{1}{G(1)} \int_{0}^{\infty} \sum_{n=1}^{n'+1} \alpha(n'-n+1,t) P_n dH(t) \qquad (13)
$$

Taking  $\sum_{n=1}^{\infty}$  after multiplying  $x^{n'}$  in both sides in equation  $n'=1$  $\overline{a}$ 

(13), and transforming into equation of the GF, equation (14) is derived.

$$
G_R(x) = \frac{1}{xG(1)} G(x)H^*(z)
$$
 (14)

where  $G_R(x) = \sum_{n=0}^{\infty} R_n x^n$  $=\sum_{n=0}^{\infty} R_n x^n$ .

Furthermore, the probability that the number of arrival data during waiting time and sending time is  $n'$  is equal to  $R_n$ , so that the following equation is obtained.

$$
R_{n'} = \int_{0}^{\infty} \alpha(n',t) dW(t) \otimes H(t) \tag{15}
$$

where  $W(t)$  is the distribution function of waiting time.

Taking  $\sum_{n=1}^{\infty}$  after multiplying  $x^{n'}$  in both sides in equation  $n'=1$  $\overline{a}$ 

(15) and transforming into equation of the GF, equation (16) is derived.

$$
G_R(x) = W^*(z)H^*(z)
$$
 (16)

where  $W^*(z)$  is the LST of  $W(t)$ .

From equation (14) and (16),  $W^*(z)$  is obtained as follows.

$$
W^*(z) = \frac{1}{G(1)x}G(x) \tag{17}
$$

When  $z = 0$  after differentiating equation (17) by *z*, the left side of equation is obtained as follows.

$$
\frac{dW^*(z)}{dz}\Big|_{z=0} = W^{*'}(0) = -w
$$
 (18a)

where *w* is a mean of  $W(t)$ .

On the other hand, the right side is obtained as follows.

$$
\frac{d}{dz} \frac{G(x)}{xG(1)} \Big|_{z=0} = \frac{d}{dx} \frac{G(x)}{xG(1)} \frac{dx}{dz} \Big|_{x=1}
$$
\n
$$
= -\frac{1}{\lambda} \frac{d}{dx} \frac{G(x)}{xG(1)} \Big|_{x=1}
$$
\n
$$
= -\frac{1}{\lambda G(1)} \frac{G'(x)x - G(x)}{x^2} \Big|_{x=1}
$$
\n
$$
= -\frac{1}{\lambda G(1)} \{G'(1) - G(1)\}
$$
\n(18b)

From equation (18a) and (18b), the mean waiting time *w* is derived.

$$
w = \frac{1}{\lambda G(1)} \{ G'(1) - G(1) \}
$$
 (19)

 $G'(1)$  has been already derived as equation (10), (11a), (11b), (12a) and (12b), so that the mean waiting time *w* can be calculated.

Fig. 7 shows a relation between arrival ratio  $\lambda$  and the mean waiting time *w*.  $h=5$ ,  $h_c=1$ , and  $H(t)$  and  $H_c(t)$  obey uniform distributions.



In Fig. 7, it is confirmed that the mean waiting time becomes long as the link probability *q* becomes low and as the arrival ratio  $\lambda$  becomes high. In the case of  $q=0.1$ , the mean waiting time becomes infinite when  $\lambda$  is over 0.069. This means that when  $q=0.1$  and  $\lambda$  is over 0.069, the sending buffer is overflow because there are more arrival data than sending data.

In the case that  $H(t)$  and  $H_c(t)$  obey exponential distributions, Fig. 8 is illustrated.



Comparing Fig. 8 with Fig. 7, the mean waiting time slightly becomes long, but Fig. 8 shows similar tendency.

Next, for Fig. 7, in the case of *h*=8, the mean waiting time is shown in Fig. 9.



Comparing Fig. 9 with Fig. 7, the mean waiting time becomes longer and it becomes infinite when  $\lambda$  is over 0.057. On comparing two figures, it is confirmed that the mean waiting time becomes longer so that sending time becomes long.

Again, for Fig. 7, in the case of  $h_c = 2$ , the mean waiting time is shown in Fig. 10.



The mean waiting time is longer than that in Fig. 7, and in the case of  $q=0.1$ , the mean waiting time becomes infinite when  $\lambda$  is over 0.045. The stand-by time does not influence the mean waiting time very much in the case of high *q*, but influences that considerably in the case of  $q=0.1$ . This means that the number of stand-by decreases in the case of high *q*, so that the time of stand-by does not influence very much.

For Fig. 9, in the case of  $h_c = 2$ , the mean waiting time is illustrated in Fig. 11.



The mean waiting time is longer than that in Fig. 9. And, like Fig. 10, the influence of the stand-by time to the mean waiting time is confirmed.

#### IV. CONCLUSION

In this paper, a characteristic of sending data is clarified in bus network. In the analysis, taking note of one station connected to the bus, the link probability on which the link from the station to a receiving station can be established is defined, and the probability of the number of waiting data, the mean of the number of waiting data at the ready of sending and the mean waiting time are derived. Furthermore, some numerical examples are illustrated and the influence of the link probability, sending time and stand-by time to the characteristics of sending data is clarified.

The characteristic of bus network in this paper is a base of considering efficiency of bus network. And, the analysis method in this paper may be applied to other networks.

The following considerations remain for further study:

- 1) Analyzing the bus network with priority.
- 2) Clarifying a relation between the link probability and the buffer size of a receiving station.

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